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Therefore, the differential equation of the curve of which these lines are tangents is

$$\left(\frac{dy}{dx}\right)^2 + \frac{b^2 - y^2 - a^2 + x^2}{xy} \left(\frac{dy}{dx}\right) - 1 = 0.$$

The solution of this equation is the conic section

$$\frac{x^2}{m^2} + \frac{y^2}{n^2} = 1,$$

or, in parameter form, $x = m \cos \phi$, $y = n \cos \phi$. From these we get

$$\frac{dy}{dx} = -\frac{n}{m} \cot \phi.$$

Substituting these values of x , y , and dy/dx in the differential equation, we get $m^2 - n^2 = a^2 - b^2$.

Therefore, the required outer curve is any one of a system of ellipses or hyperbolas confocal with the inner curve.

(b) If the inner curve is an hyperbola, we have the same result.

(c) If the inner curve is the parabola, $y^2 = 4p(x+p)$.

Proceeding in the same way, the equation of the tangents is

$$(y^2 - 4px - 4p^2)(y'^2 - 4px' - 4p^2) - [yy' - 2p(x+x') - 4p^2] = 0.$$

The differential equation is

$$\left(\frac{dy}{dx}\right)^2 + \frac{2x}{y} \left(\frac{dy}{dx}\right) - 1 = 0.$$

The solution is, $y^2 = 4c(x+c)$.

This is a system of parabolas confocal with the given one.

Also solved by the Proposer.

MECHANICS.

222. Proposed by W. J. GREENSTREET, Stroud, England.

Find the maximum angle of inclination to the line of greatest slope of a uniform rod resting on a rough inclined plane and capable of turning freely round a point on it.

Solution by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

By the maximum angle is evidently meant the angle of limiting equi-

librium. Let R = the reaction of the rod concentrated at G , its center of gravity; μ = coefficient of friction; W = weight of rod; β = inclination of plane to horizon; θ = required angle.

The reaction R at G is perpendicular to the plane, and is $R = W \cos \beta$; while $W \sin \beta$ is the pull down the plane. Now as friction acts opposite to the direction of motion, we get $W \sin \beta \sin \theta = \mu R$.

$$\therefore W \sin \beta \sin \theta = \mu W \cos \beta, \text{ or } \sin \theta = \mu \cot \beta; \theta = \sin^{-1}(\mu \cot \beta).$$

223. Proposed by G. B. M. ZERR, A. M., Ph. D., Philadelphia, Pa.

A sphere, radius $r = \frac{1}{8}$ inches, density $\delta = 11.38$, falls from a height $h = 500$ feet, into a lake depth $l = 40$ feet. Find time of falling to surface of lake, time of falling from surface of lake to bottom, and total time of falling. Also the velocity at the bottom.

Solution by the PROPOSER.

Let $r = \frac{1}{8}$ inch = $\frac{1}{8}$ feet = radius of ball; $g = 32.2$ = gravity; $e_2 = g$ = weight of unit volume of water; δ = density of air = .001293; $e_1 = g \delta$ = weight of unit volume of air; $R_1 v^2$ = resistance of air; $R_2 v^2$ = resistance of water; A = greatest sectional area of sphere = πr^2 ; k = a constant = 0.51 for the sphere; m = mass of sphere = $\frac{1}{8}g$; v_1 = velocity at surface of lake; v_2 = velocity at bottom of lake. Then the equations of motion are

$$m \frac{dv}{dt} = mg - Rv^2 \dots (1), \quad mv \frac{dv}{dx} = mg - Rv^2 \dots (2).$$

From (1), $t = m \int \frac{dv}{mg - Rv^2}$. For the time, T , from rest to the surface of the lake, $R = R_1$, and the limits of v are 0 and v_1 .

$$\therefore T = \frac{1}{2} \sqrt{\frac{m}{gR_1}} \log \left[\frac{\sqrt{(mg) + \sqrt{(R_1)v_1}}}{\sqrt{(mg) - \sqrt{(R_1)v_1}}} \right] \dots (3).$$

For the time, T_1 , from the surface to the bottom of the lake, $R = R_2$, and the limits of v are v_1 and v_2 .

$$\therefore T_2 = \frac{1}{2} \sqrt{\frac{m}{gR_2}} \log \left[\frac{[\sqrt{(mg) + \sqrt{(R_2)v_2}}][\sqrt{(mg) - \sqrt{(R_2)v_1}}]}{[\sqrt{(mg) - \sqrt{(R_2)v_2}}][\sqrt{(mg) + \sqrt{(R_2)v_1}}]} \right] \dots (4).$$

$$\text{From (2), } x = m \int \frac{v dv}{mg - Rv^2}.$$

Between the limits v and V we get for x ,